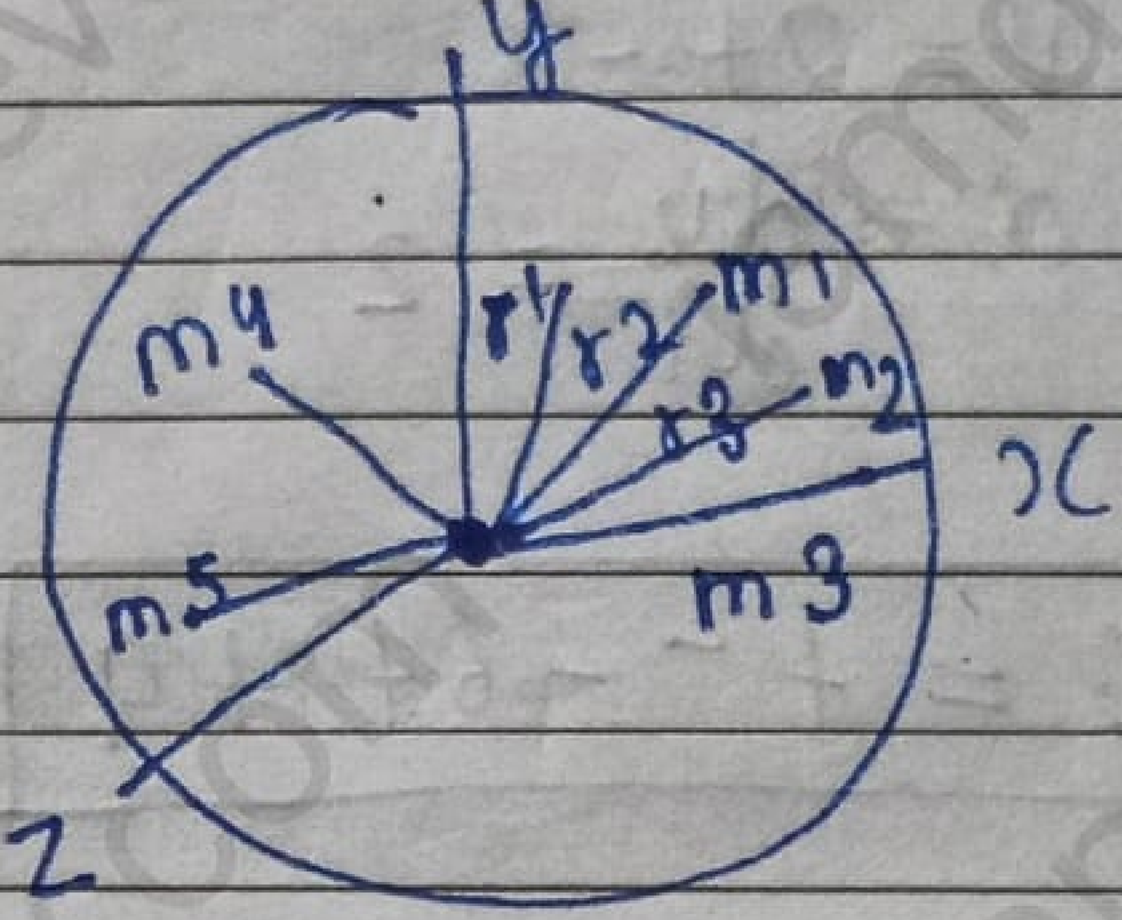
~~Ques Define Rigi~~

Rotational Motion



$M = m_1 + m_2 + m_3 - - - - . m_n$

$\vec{R}_{C.O.M} = \vec{R}_{C.M.} = m_1\vec{r_1} + m_2\vec{r_2}$

$\vec{R}_{CM} = \sum_{i=1}^{n} = \frac{M_i r_i}{M}$

$\vec{R} = \vec{x}\hat{i} + \vec{y}\hat{j} + z\hat{k}$

$\vec{R}_{C.M.} = \vec{x}_{cm}\hat{i} + \vec{y}\hat{j} + z_{cm}\hat{k}$

$\vec{R}_{CM} = \sum_{i=1}^{n} \frac{m_i\vec{r_i}}{M}$

$\vec{x}_{CM} = \sum_{=1}^{n} = \frac{M_i\vec{x_i}}{M}$

$\vec{z}_{CM} = \sum_{i=1}^{n} \frac{M_i z_i}{M}$

→ Relation between power, force and velocity

Because we know that power

$$P = \frac{dW}{dt} \quad -(1)$$

"Rate change of work"

$$\text{But } W = FS \quad -(2)$$

$$P = \frac{d(FS)}{dt} = F = \frac{d(S)}{dt} = FV$$

→ Relation between power and Torque

$W = FS$

In ~~Rotational~~ Motion

~~Required~~ Work done for small $d\theta$ $= dW = \tau \, d\theta$

$$\frac{dW}{dt} = \tau \frac{d\theta}{dt}$$

$$\frac{dW}{dt} = \tau \frac{d\theta}{dt}$$

$$P = \tau\omega$$

Angular momentum of a body about a given axis is the product of Linear momentum & perpendicular distance of line of action of Linear momentum from the axis of rotation it is always represent by 'L'

$$\vec{L} = \vec{r} \times \vec{p}$$

$\vec{L}$ → Angular momentum $\vec{p}$ → Linear momentum

Vector Quantity
$m\,kg\,\frac{m}{s} = kg\,\frac{m^2}{s}$

Ques - To maintain a Rotor at a uniform angular speed $200 s^{-1}$, an engine need to transmit a torque of 180 N/m. What is the power of engine required?

$\omega = 200/sec.$
$\tau = 180\ Nm$
$P = ?$

$P = \tau\omega$
$= 180 \times 200$
$= 36000$ Watt

→ Relation between angular momentum & Torque

Prove that $\frac{d}{dt}L = \tau \Rightarrow \frac{dL}{dt} = \tau$

$$\frac{d}{dt}(A \times B)$$

$$= \frac{dA}{dt} \times B + A \times \frac{dB}{dt}$$

Because we know that from the relation between L & P

$$\vec{L} = \vec{r} \times \vec{P}$$

$$\frac{d}{dt}\tau = \frac{d}{dt}(\vec{r} \times \vec{P})$$

$$= \frac{d}{dt}\vec{r} \times \vec{P} + \vec{r} \times \frac{d\vec{P}}{dt}$$

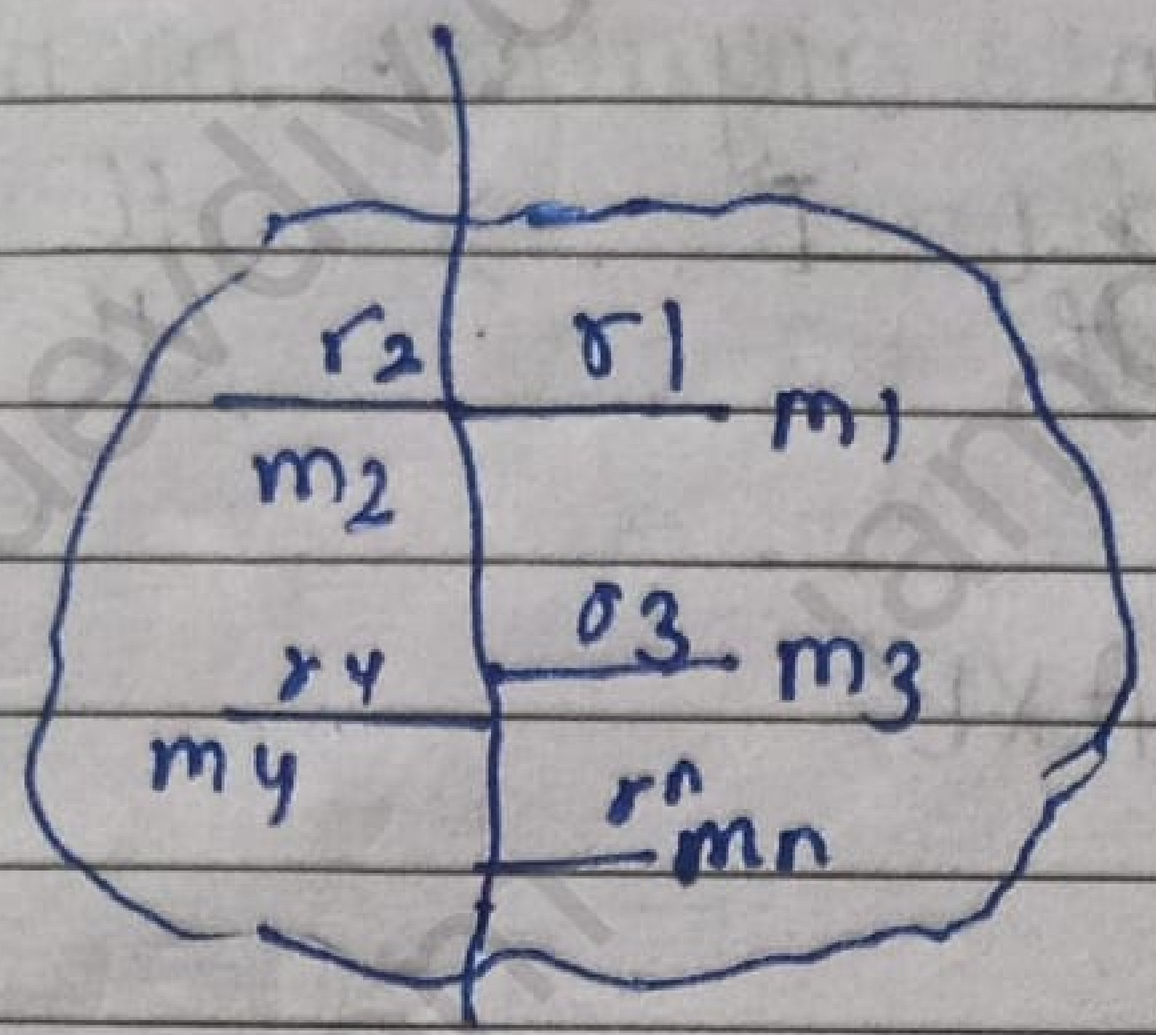
$$= \frac{d}{dt}\vec{r} \times m\vec{r} + \vec{r} \times \vec{F}$$

$$= \frac{d\vec{L}}{dt} = \vec{v} \times m\vec{v} + \vec{\tau}$$

$$= \frac{dL}{dt} = (\vec{v} \times \vec{v})m + \vec{\tau}$$

$$= \frac{d\vec{L}}{dt} = \vec{\tau} + 0$$

$$= \frac{dL}{dt} = \vec{\tau}$$



\# $\tau = I\alpha \rightarrow$ Angular accelaration
τ Torque, I Moment of Inertia

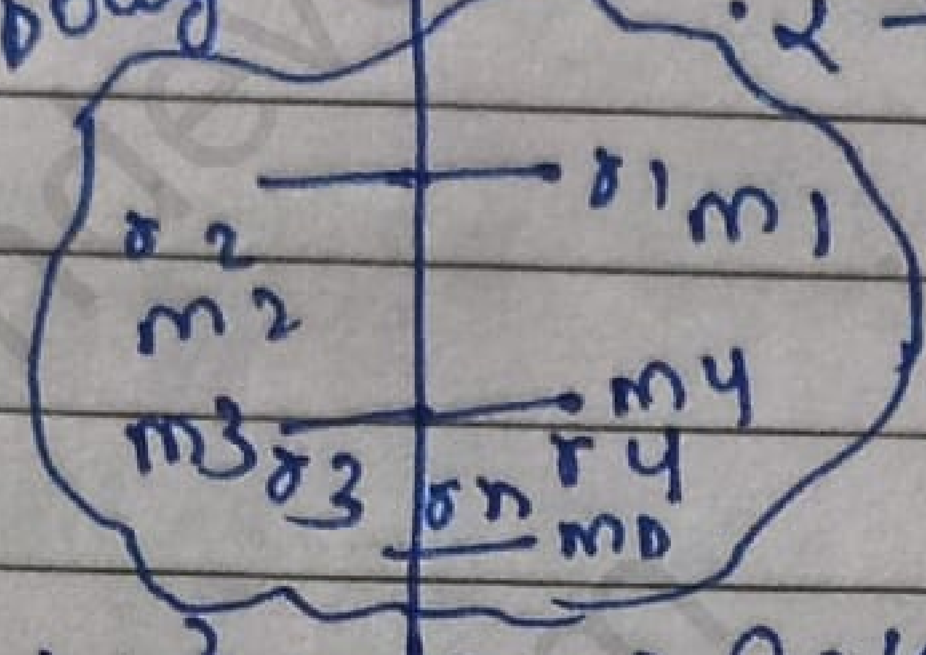
Let us consider a rigid body with their n particles their masses are $m_1, m_2, m_3, m_4 - - - - m_n$ and perpendicular distances of particle along the rotational axis $r_1, r_2, r_3 - - - - r_n$. This rigid body rotate along to rotational axis with angular acceleration α because this is a rigid body so angular acceleration α will be constant for all every elementary particles of the body. Suppose that their working ~~on their~~ force on particles are $f_1, f_2, f_3 - - - - - f_n$. Now from the above diagram net Torque on the rigid body.

$$\tau = \tau_1 + \tau_2 + \tau_3 - - - - + \tau_n$$
$$= r_1 \times f_1 + r_2 \times f_2 + r_3 \times f_3 - - - + r_n \times f_n$$
$$= r_1 m_1 a_1 + r_2 m_2 a_2 + r_3 m_3 a_3 + - - - + r_n m_n a_n$$
$$= r_1 m_1 r_1 \alpha + r_2 m_2 r_2 \alpha - - - - - + r_n m_n r_n \alpha$$
$$\tau = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + m_3 r_3^2 \alpha + - - - m_n r_n^2 \alpha$$
$$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 - - - - - - - - + m_n r_n^2)\alpha$$
$$= (I_1 + I_2 + I_3 - - - - I_n)\alpha$$
$$= \tau = I\alpha$$

Moment of Inertia

Rigidbody A ω → Angular velocity

α → Angular acceleration



B → Rotational axis

$I_2 = m_2 r_2^2$

$I_3 = m_3 r_3^2$

$I_4 = m_4 r_4^2$

$I_1 = m_1 r_1^2$

$M = m_1 + m_2 + m_3 + m_4 - - - - m_n$

$I = m_1 r_1^2 + m_2 r_2^2 + - - - m_n r_n^2$

Unit of $I = kg\,m^2$

Dimensional formula $= [ML^2]$

$\rightarrow$ $L = I\,\omega$

moment of Inertia

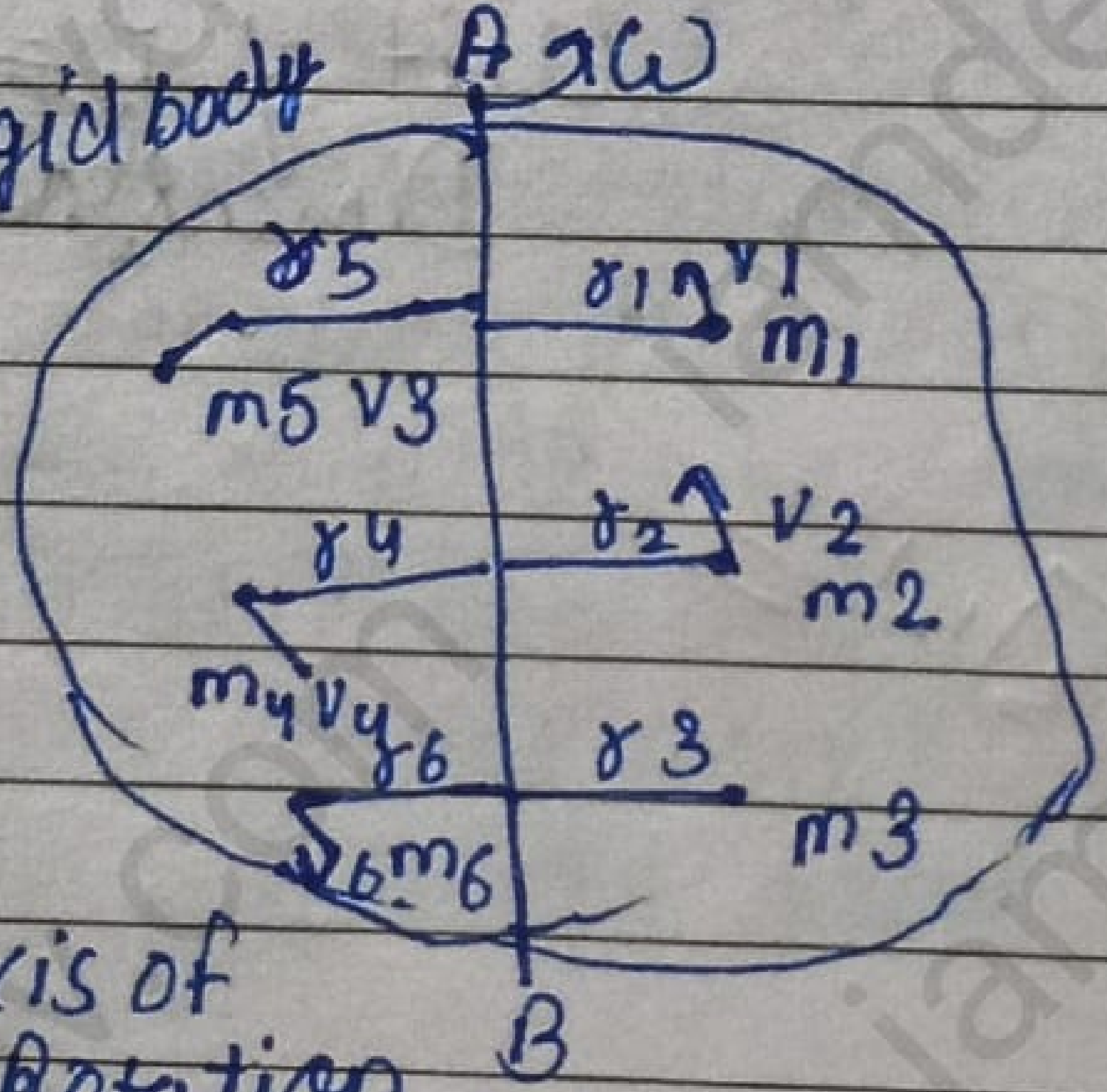
angular velocity

angular momentum

$$I = Mr^2$$
$$L = \vec{r} \times \vec{p}$$
$$p = mv$$
$$v = r\omega$$

$$L = \vec{r} \times \vec{p}$$
$$= \vec{r} \times \vec{mv}$$
$$= rmr\omega$$
$$= mr^2\omega$$
$$\boxed{L = I\,\omega}$$

☆



AB = Axis of Rotation

- ~~From the above diagram~~ Let us consider a rigid body with n particles their mass $m_1, m_2 - - - . m_n$
- The perpendicular distances along to the rotational axis are $r_1, r_2, r_3 - - - . r_3$ their respective
- If their indivision angular moment $L_1, L_2, L_3 - - L_n$

Now $L = L_1 + L_2 + L_3 + L_4 \cdots\cdots L_n$

$= r_1 \times P_1 + r_2 \times P_2 + r_3 \times P_3 + \cdots\cdots r_n \times P_n$

$(\because P = mv)$

$= r_1 \times m_1 v_1 + r_2 \times m_2 v_2 + r_2 m_3 v_3 + \cdots\cdots + r_n m_n v_n$

$(\because V = r\omega)$

$= r_1 m_1 r_1 \omega + r_2 m_2 r_2 \omega \cdots\cdots + r_n m_n r_n$

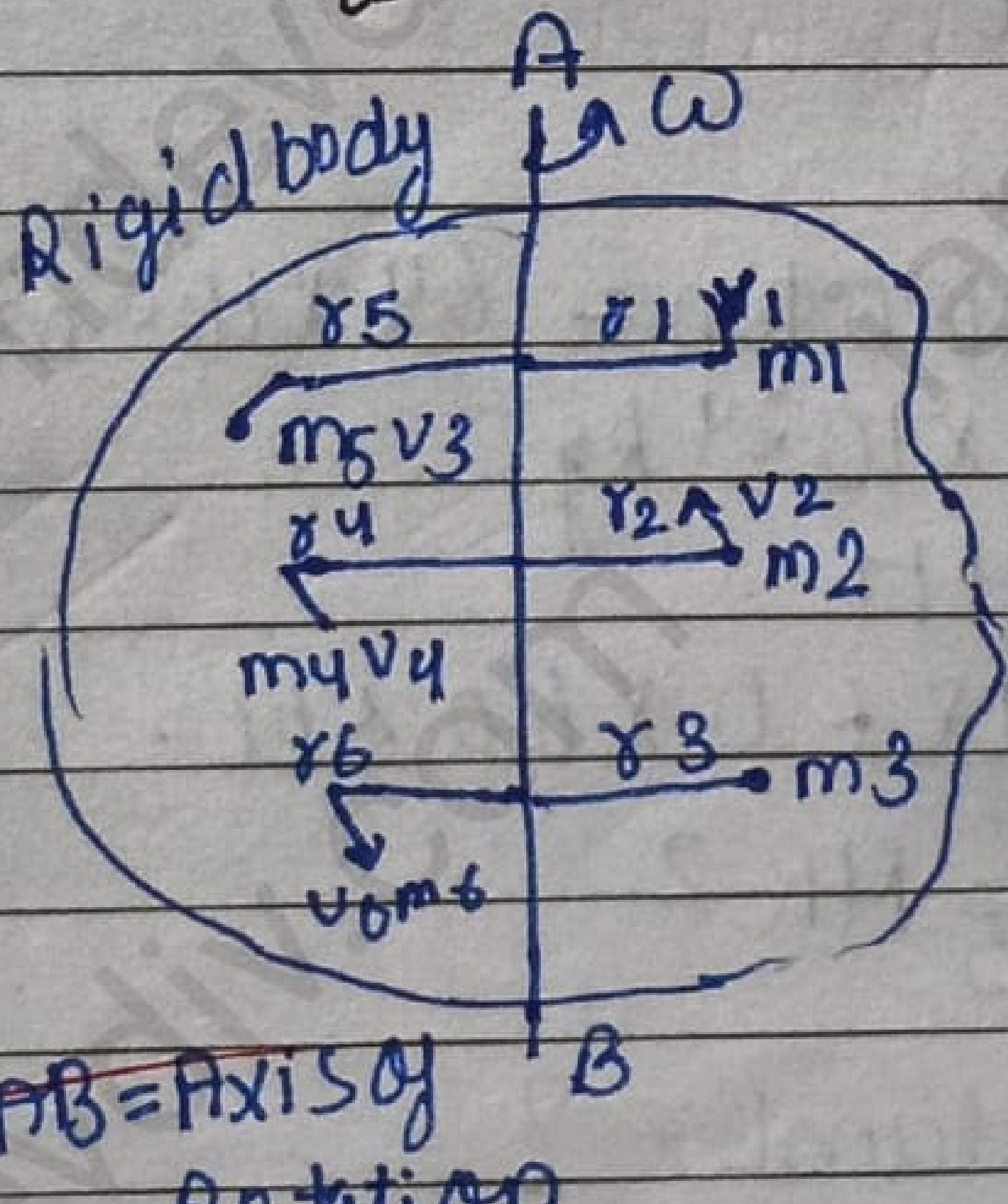
$= m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_3 r_3^2 \omega \cdots\cdots + m_n r_n^2 \omega$

$= (m_1 r_1^2 + m_2 r_2^2 + \cdots\cdots + m_n r_n^2)\omega$

$= (I_1 + I_2 + I_3 \cdots\cdots + I_n)\omega$

$\boxed{L = I\omega}$

→ Prove that $K.E. = \frac{1}{2} I\omega^2$



Now $K = K_1 + K_2 + K_3 + K_4 + \cdots\cdots K_n$

$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2 \cdots\cdots \frac{1}{2} m_n v_n^2$

$= \frac{1}{2} m_1 r_1 \omega_1^2 + \frac{1}{2} m_2 r_2 \omega_2^2 + \frac{1}{2} m_3 r_3 \omega_3^2 \cdots\cdots \frac{1}{2} m_n r_n \omega_n^2$

$= (\frac{1}{2}\omega^2 [m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \cdots\cdots m_n r_n^2]$

$= \frac{1}{2}\omega^2 (I_1 + I_2 + I_3 + \cdots\cdots I_n)$

$$\boxed{K.E. = \frac{1}{2} I\omega^2}$$

~~Prove that rate change of angular momentum?~~

Ques Prove that rate change of angular momentum?

$$\boxed{\frac{dL}{dt} = \tau}$$

$L = I\omega$

$\frac{dL}{dt} = I\frac{d\omega}{dt}$

$\cancel{\frac{dL}{dt} = I\alpha}$ $\frac{dL}{dt} = I\alpha$

Earth

$R = 6400\ km = 64 \times 10^5\ m$

$M = 6 \times 10^{24}\ kg$

$T = 24H$

$= 24 \times 60 \times 60\ sec.$

~~I about axis~~ $= \frac{2}{5} MR^2$

Angular momentum $L = ?$

$I\omega$

$= \frac{2}{5} MR^2 \times \frac{2\pi}{T}$

$= \frac{2}{5} \times 6 \times 10^{24} = (64 \times 10^5)^2 \times \frac{2\pi}{24 \times 60 \times 60}$

12

6

$= 10^{24}$

$\cancel{512}$ $\cancel{256}$ 128
$\cancel{1024}$
$\cancel{2048}$

$$\frac{10^{24}}{5} \times \cancel{4096} \times 10^{10} \times \frac{\pi}{\cancel{60} \times \cancel{60}}$$

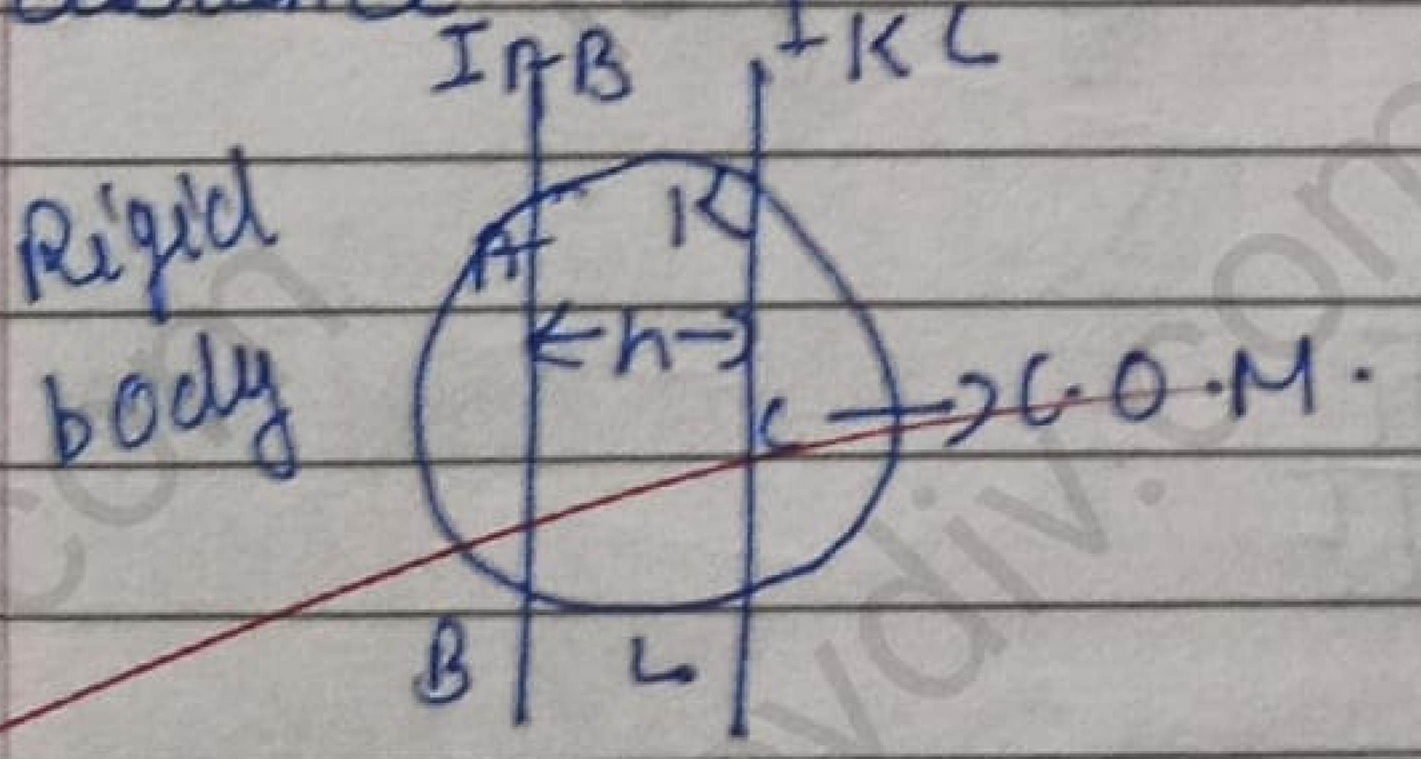
$\cancel{30}$ $\cancel{30}$ 15
15

$$= \frac{10^{34} \times 128\pi}{5 \times 15 \times 15} = \frac{10^{34} \times 128 \times 22}{5 \times 15 \times 15 \times 7}$$

$$= 4.5784 \times 10^{34}$$

Theorem of moment of Inertia

Parallel axis theorem:- Acc. to the theorem of parallel axis, ~~moment~~ moment of inertia of a rigid body about any axis AB is equal to moment of inertia of the body about another axis KL passing through centre of mass C of the body in a direction parallel to AB, plus the product of total mass, m of the body and square of perpendicular distance

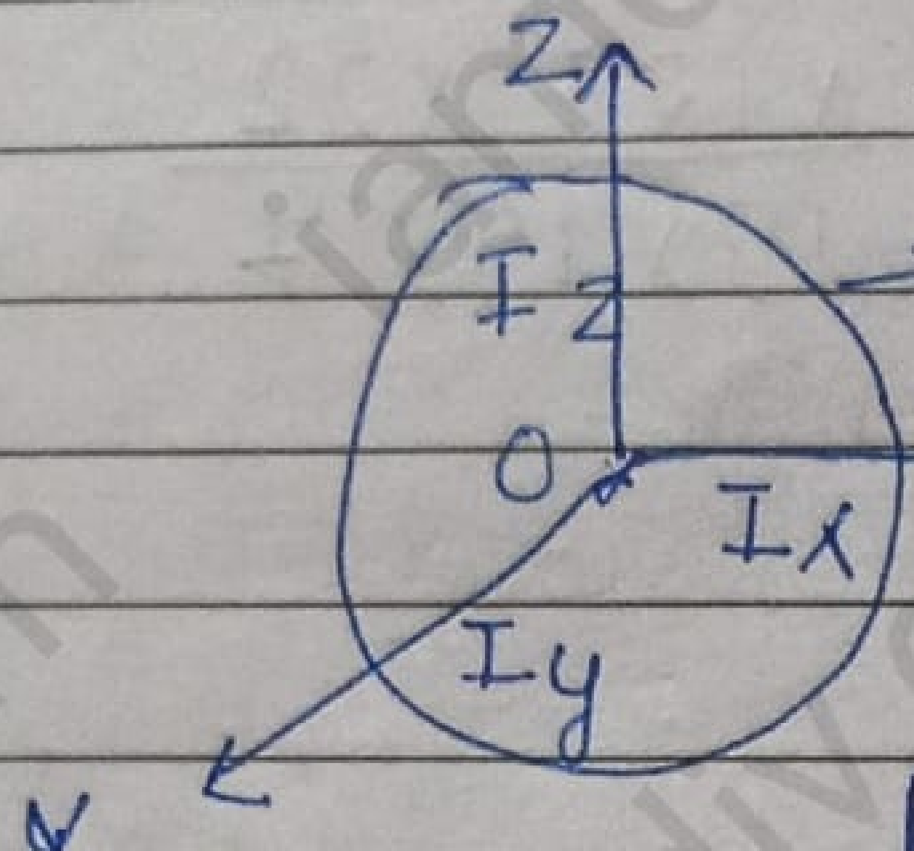


$$\boxed{I_{AB} = I_{KL} + Mh^2}$$

M = Total mass of body
AB = Axis of Rotation
$\boxed{AB \parallel KL}$

→ Perpendicular axis theorem:- Acc. to this theorem of perpendicular axis, the moment of inertia of plane lamina about any axis $O_Z \perp$ to the plane of lamina is equal to sum of the moment of inertia of the lamina about any 2 mutually ⊥ axis, in the lamina Ox & Oy in plane of the lamina

Ox, Oy, Oz All are meeting at point O



XYZ -plane

→ Rigid body

3d lamina

3d. plane

$\boxed{I_Z = I_X + I_Y}$ or $\boxed{I_{OZ} = I_{OX} + I_{OY}}$

Principle of Conservation of Angular Momentum:-

According to this rule, when no external force/Torque act on a system of particles then the total angular momentum of system remains always a constant

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\text{if } \vec{\tau} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0$$

$$d\vec{L} = 0$$

$$\vec{L} = \text{Constant}$$